

# 1 Simply typed $\lambda$ -calculus

## Language definition

$e ::= x \mid \lambda x: T. e \mid e e$  (expressions)  
 $v ::= \lambda x: T. e$  (values)  
 $T ::= T \rightarrow T$  (types)  
 $\Gamma ::= \cdot \mid \Gamma[x: T]$  (type env.)

## Typing rules $\boxed{\Gamma \vdash e: T}$

$$\begin{array}{c} \text{(t-var)} \frac{\Gamma(x) = T}{\Gamma \vdash x: T} \qquad \text{(t-abs)} \frac{\Gamma[x: T_1] \vdash e: T_2}{\Gamma \vdash \lambda x: T. e: T_1 \rightarrow T_2} \\ \\ \text{(t-abs)} \frac{\Gamma \vdash e_1: T_1 \rightarrow T_2 \quad \Gamma \vdash e_2: T_1}{\Gamma \vdash e_1 e_2: T_2} \end{array}$$

## Evaluation rules $\boxed{e \longrightarrow e'}$

$$\begin{array}{c} \text{(e-app1)} \frac{e_1 \longrightarrow e'_1}{e_1 e_2 \longrightarrow e'_1 e_2} \qquad \text{(e-app2)} \frac{e_2 \longrightarrow e'_2}{e_1 e_2 \longrightarrow e_1 e'_2} \\ \\ \text{(e-appabs)} \frac{}{(\lambda x: T. e)v \longrightarrow [v/x]e} \end{array}$$