

1 Simply typed λ -calculus

Language definition

$$\begin{aligned} e &::= x \mid \lambda x : T. e \mid e e && \text{(expressions)} \\ v &::= \lambda x : T. e && \text{(values)} \\ T &::= T \rightarrow T && \text{(types)} \\ \Gamma &::= \cdot \mid \Gamma[x : T] && \text{(type env.)} \end{aligned}$$

Typing rules $\boxed{\Gamma \vdash e : T}$

$$\begin{aligned} \text{(t-var)} \quad & \frac{\Gamma(x) = T}{\Gamma \vdash x : T} & \text{(t-abs)} \quad & \frac{\Gamma[x : T_1] \vdash e : T_2}{\Gamma \vdash \lambda x : T. e : T_1 \rightarrow T_2} \\ & & \text{(t-abs)} \quad & \frac{\Gamma \vdash e_1 : T_1 \rightarrow T_2 \quad \Gamma \vdash e_2 : T_1}{\Gamma \vdash e_1 e_2 : T_2} \end{aligned}$$

Evaluation rules $\boxed{e \longrightarrow e'}$

$$\begin{aligned} \text{(e-app1)} \quad & \frac{e_1 \longrightarrow e'_1}{e_1 e_2 \longrightarrow e'_1 e_2} & \text{(e-app2)} \quad & \frac{e_2 \longrightarrow e'_2}{e_1 e_2 \longrightarrow e_1 e'_2} \\ \text{(e-appabs)} \quad & \frac{}{(\lambda x : T. e)v \longrightarrow [v/x]e} \end{aligned}$$