An introduction to Logic Programming

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In this lecture ...

- What is Logic Programming?
- A bit of history
- Facts and Rules
- Queries
- Unification
- Resolution of queries
- Negation as failure
- 8-queens problem

Logic Programming?

- **Use of mathematical logic for programming**
- **Using logic as:**
  - representational language (data)
  - procedural language (control)

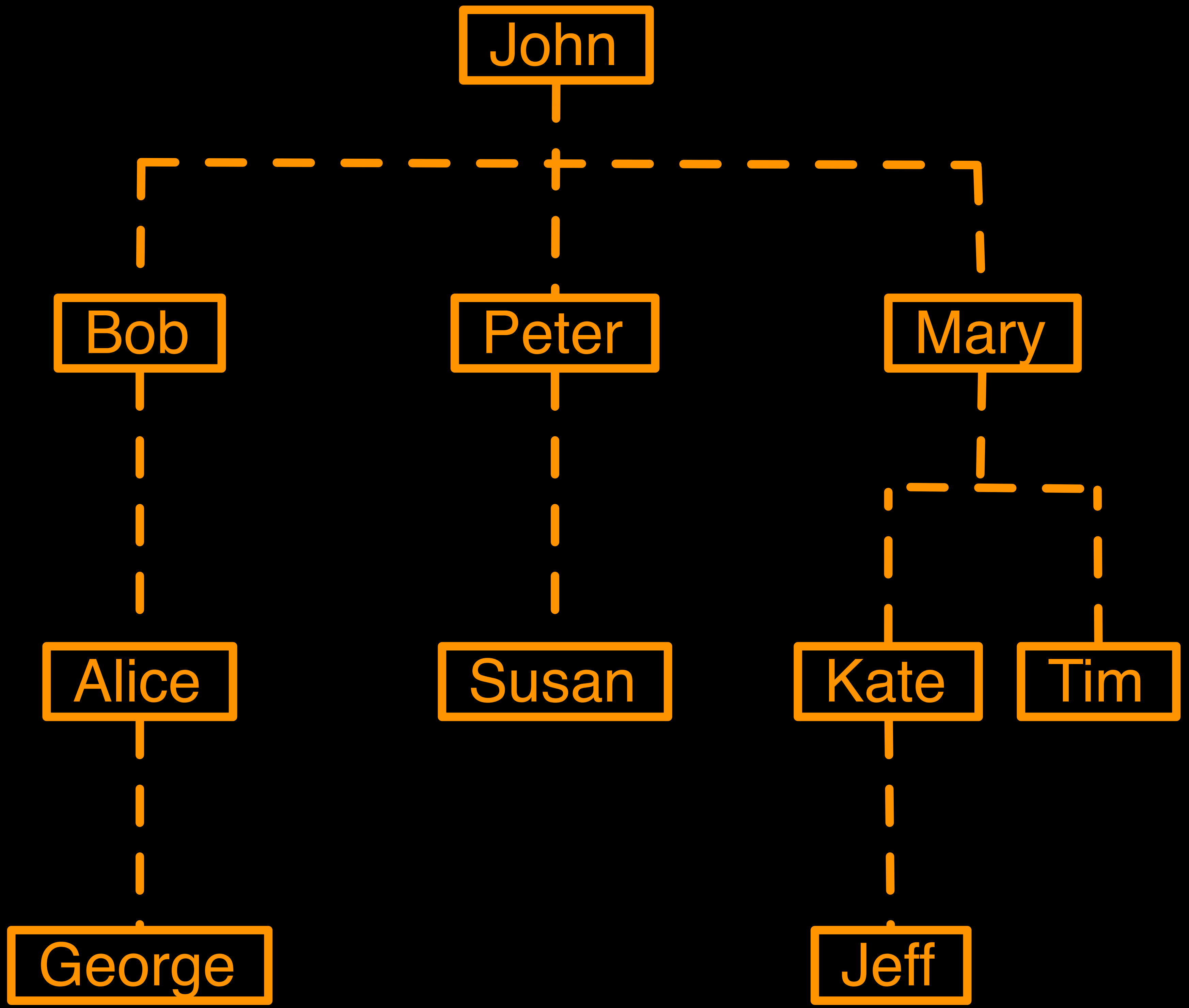
If there is an emergency
then call the police

- **Declarative language**
  - Specify what, not how

A bit of history ...

- **Artificial Intelligence (1960’s, early 1970’s):**
  - Research about knowledge representation and inference
  - Theorem-provers
  - Problem-solvers
  - Natural language processing
- **Prolog (1972, Colmerauer)**
  - Dual declarative/procedural interpretation
  - Turing complete programming language
  - Focus of this lecture
- **Many “flavours”:**
  - datalog
  - forward chaining vs. backward chaining

An example: family tree



Facts and rules

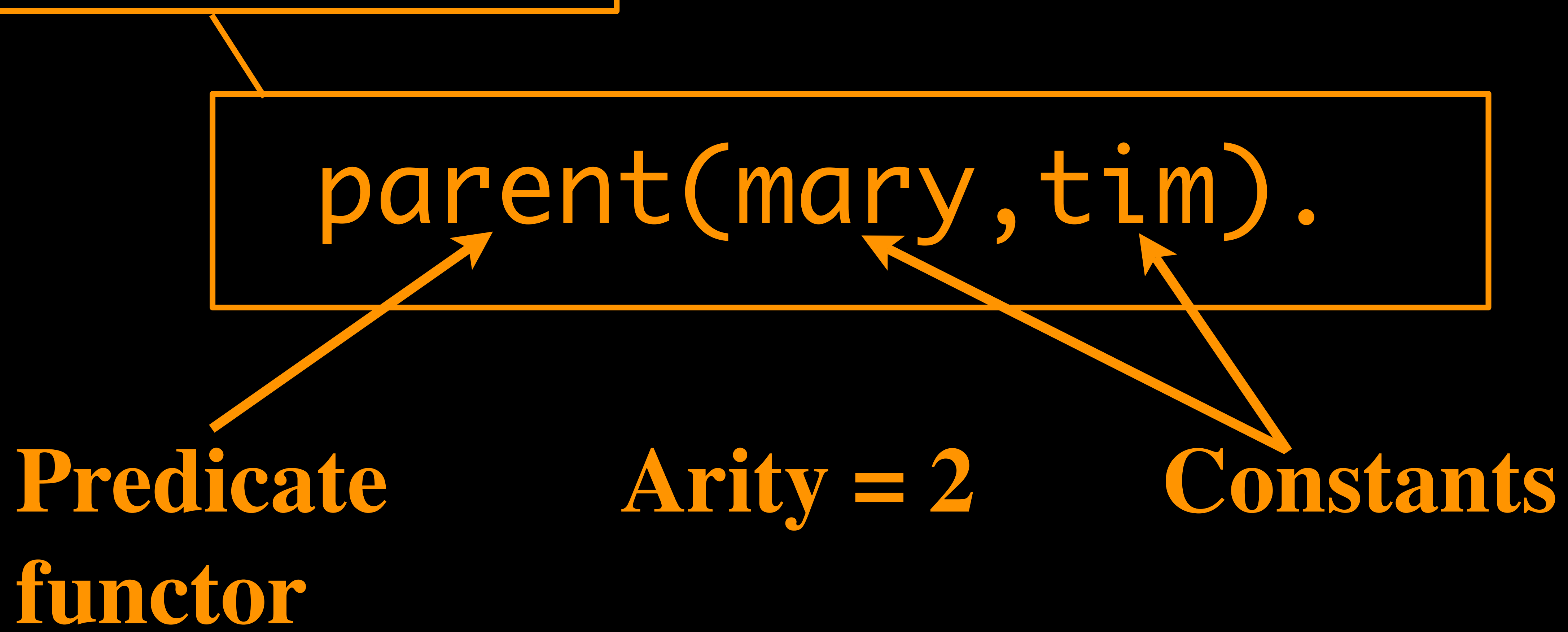
- **Logic programming is about relationships**
- **Two main concepts:**
  - Facts expressing your basic relationships
  - Rules expressing derivable knowledge
- **In our example:**
  - Facts: parent relationships, female/male
  - Rules: brother, sister, grandparent, sibling, niece ...

Facts

```
male(john).
parent(john,bob).
parent(john,peter).
parent(john,mary).
female(mary).
parent(mary,kate).
parent(mary,tim).
```

Facts

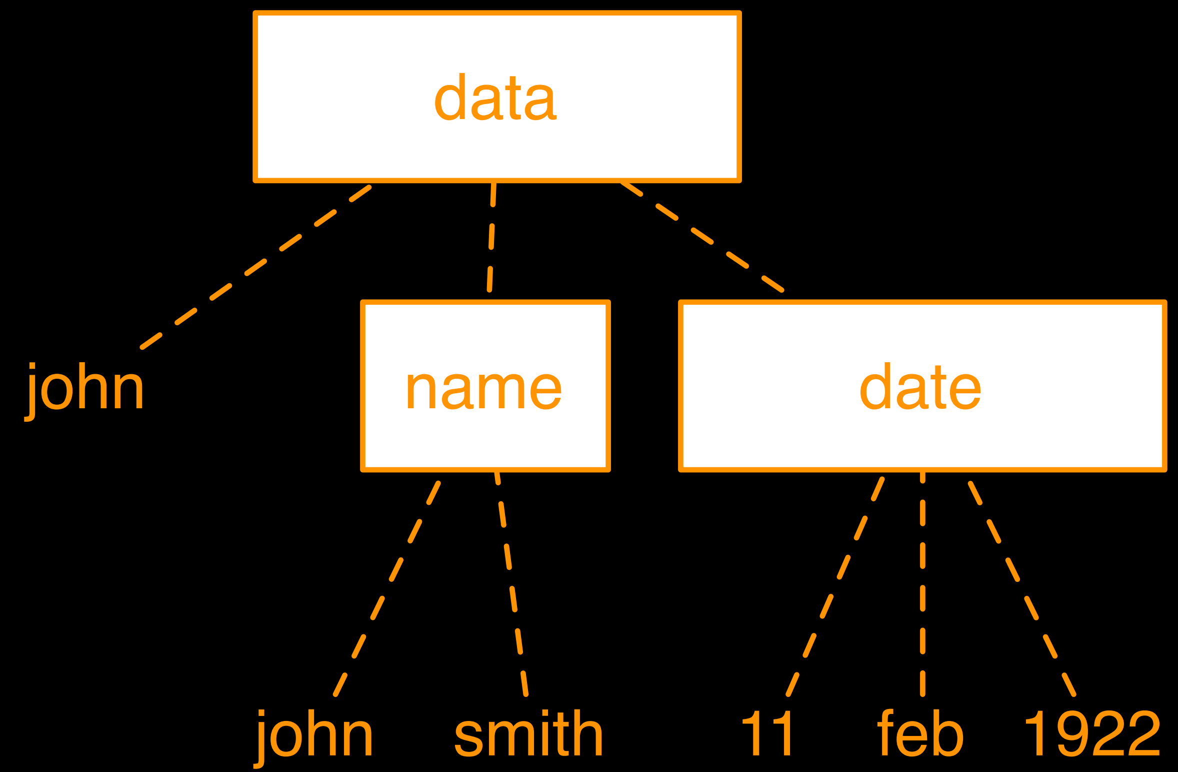
```
male(john).
parent(john,bob).
parent(john,peter).
parent(john,mary).
female(mary).
parent(mary,kate).
parent(mary,tim).
```



More structured data

```
data(john,
     name(smith,john),
     date(11,feb,1922))
```

functor



Rules

```
grandparent(Grandparent, Grandchild) :-
    parent(Grandparent, Parent),
    parent(Parent, Grandchild)
```

Rules

Conclusion

```
grandparent(Grandparent, Grandchild) :-
    parent(Grandparent, Parent),
    parent(Parent, Grandchild)
```

Rules

Conclusion **Variables**

```
grandparent(Grandparent, Grandchild) :-
    parent(Grandparent, Parent),
    parent(Parent, Grandchild)
```

Rules

Conclusion **Variables** **if**

```
grandparent(Grandparent, Grandchild) :-
    parent(Grandparent, Parent),
    parent(Parent, Grandchild)
```

Rules

Conclusion

Variables

if

```
grandparent(Grandparent, Grandchild) :-
    parent(Grandparent, Parent),
    parent(Parent, Grandchild)
```

Conditions

Rules

Conclusion — Variables — if

```
grandparent(Grandparent, Grandchild) :-
    parent(Grandparent, Parent),
    parent(Parent, Grandchild)
```

and

Conditions

Recursion

```
ancestor(Ancestor, Child) :-
    parent(Parent, Child),
    ancestor(Ancestor, Parent)
```

Recursion

```
ancestor(Ancestor, Child) :-
    parent(Parent, Child),
    ancestor(Ancestor, Parent)
```

Recursion

```
ancestor(Ancestor, Child) :-
    parent(Parent, Child),
    ancestor(Ancestor, Parent)
```

Is this rule complete?

Multiple rules for same predicate

```
ancestor(Ancestor, Child) :-
    parent(Ancestor, Child)

ancestor(Ancestor, Child) :-
    parent(Parent, Child),
    ancestor(Ancestor, Parent)
```

Queries

- **How to perform computations?**
  - By asking queries to the Prolog engine
- **Is John the parent of Bob?**
- **Is Bob the father of Alice?**
- **Who is the father of Peter?**
- **Who are the grandchildren of John?**

- **Queries return either true/false, or a set of bindings that are a valid result**

Example queries (1)

```
Welcome to SWI-Prolog (Multi-threaded, 32 bits, Version 5.6.62)
Copyright (c) 1990-2008 University of Amsterdam.
SWI-Prolog comes with ABSOLUTELY NO WARRANTY. This is free software,
and you are welcome to redistribute it under certain conditions.
Please visit http://www.swi-prolog.org for details.

For help, use ?- help(Topic). or ?- apropos(Word).

?-
```

```
-? parent(john, bob).
true.


-? parent(john, kate).
false.
```

Queries (2)

```
-? parent(Parent, kate).
Parent = mary.


-? grandparent(john, Grandchild).
Grandchild = alice ;
Grandchild = susan ;
Grandchild = kate ;
Grandchild = tim.
```

Multi-way predicates (1)

- **Same predicate can be used to verify relations or to query relations**

```
-? ancestor(john, kate).
   true.
-? ancestor(john, Who).
   Who = bob ;
   Who = peter ;
   Who = mary ;
   Who = alice ;
   Who = susan ;
   Who = kate ;
   Who = tim ;
   Who = george ;
   Who = jeff ;
```

Multi-way predicates (2)

```
-? ancestor(Ancestor, Child).
    Ancestor = john,
    Child = bob ;
    Ancestor = john,
    Child = peter ;
    Ancestor = john,
    Child = mary ;
    Ancestor = bob,
    Child = alice ;
    Ancestor = peter,
    Child = susan ;
    Ancestor = mary,
    Child = kate ;
    Ancestor = mary,
    Child = tim ;
    ....
```

Remember!

```
ancestor(Ancestor, Child) :-
    parent(Ancestor, Child)
ancestor(Ancestor, Child) :-
    parent(Parent, Child),
    ancestor(Ancestor, Parent)
```

How does this work?

- **No magic!**
- **Prolog = inference engine**
- **Two important concepts:**
  - Unification:

    match two different entities (=)

    bind variables
  - Resolution:

    starting from a query (a goal)

    simplify this goal until we

    can find a matching basic fact

    or we can refute the goal.

Unification (1)

- **In Prolog: unify two values using =**

```
-? a = b.
false

-? a = a.
 true
```

Equality of two constants

```
-? X = a.
 X = a
```

If a variable and a constant; bind the variable

Unification (2)

```
-? data(john,name(smith,john), date(11,feb,1922))
      = data(john, Name, Birthday).
Name = name(smith, john)
Birthday = date(11, feb, 1922)
```

Structural match

```
-? data(Person,name(smith,john), Birthday)
      = data(john, Name, date(11,feb,1922).
Person = john
Name = name(smith, john)
Birthday = date(11, feb, 1922)
```

Unification (3)

```
-? X = Y, Y = a.
 X = a
 Y = a

-? X = Y, Z = Y
 X = Z
 Y = Z

-? X = a, X = b
 false
```

Unification with variables

Variables only bound once!

Resolution

```
ancestor(Ancestor, Child) :-
    parent(Ancestor, Child)
ancestor(Ancestor, Child) :-
    parent(Parent, Child),
    ancestor(Ancestor, Parent)
```

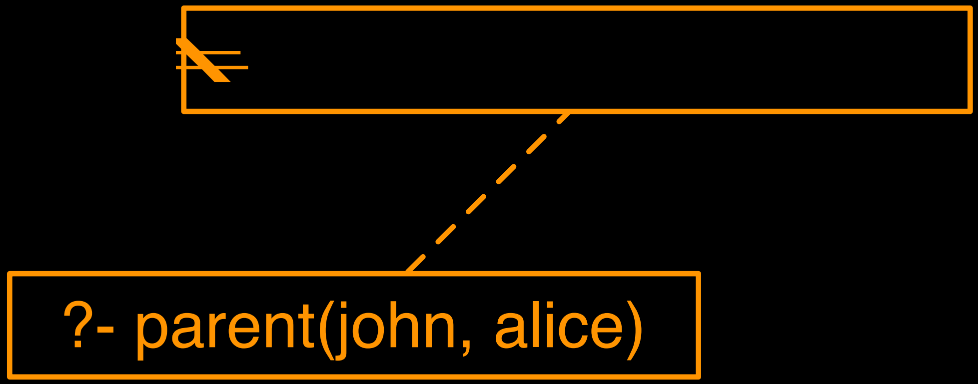
- **How do queries work?**

Bindings

Resolution

```
ancestor(Ancestor, Child) :-
      parent(Ancestor, Child)
ancestor(Ancestor, Child) :-
      parent(Parent, Child),
      ancestor(Ancestor, Parent)
```

- **How do queries work?**



Bindings

```
Ancestor = john
Child = george
```

Resolution

```
ancestor(Ancestor, Child) :-
      parent(Ancestor, Child)
ancestor(Ancestor, Child) :-
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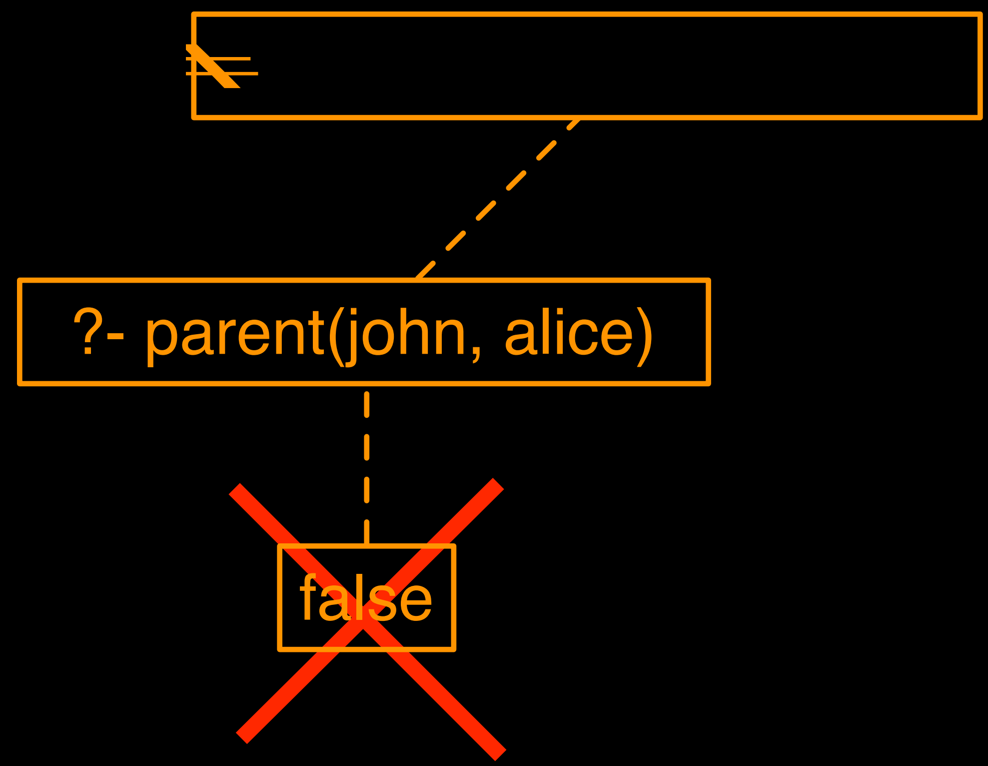

John is not the parent of Alice

Bindings

```
Ancestor = john
Child = george
```

Resolution

```
ancestor(Ancestor, Child) :-
        parent(Ancestor, Child)
ancestor(Ancestor, Child) :-
        parent(Parent, Child),
        ancestor(Ancestor, Parent)
```

- **How do queries work?**

?- parent(john, alice)

?- parent(Parent, alice),
ancestor(john, Parent)

false

John is not the parent of Alice

Bindings

```
Ancestor = john
Child = george
```

Resolution

```
ancestor(Ancestor, Child) :-
        parent(Ancestor, Child)
ancestor(Ancestor, Child) :-
        parent(Parent, Child),
        ancestor(Ancestor, Parent)
```

- **How do queries work?**

?- parent(john, alice)

false

John is not the parent of Alice

?- parent(Parent, alice),
ancestor(john, Parent)

?- ancestor(john, bob)

Bindings

```
Ancestor = john
Child = george
Parent = bob
```

Resolution

```
ancestor(Ancestor, Child) :-
        parent(Ancestor, Child)
ancestor(Ancestor, Child) :-
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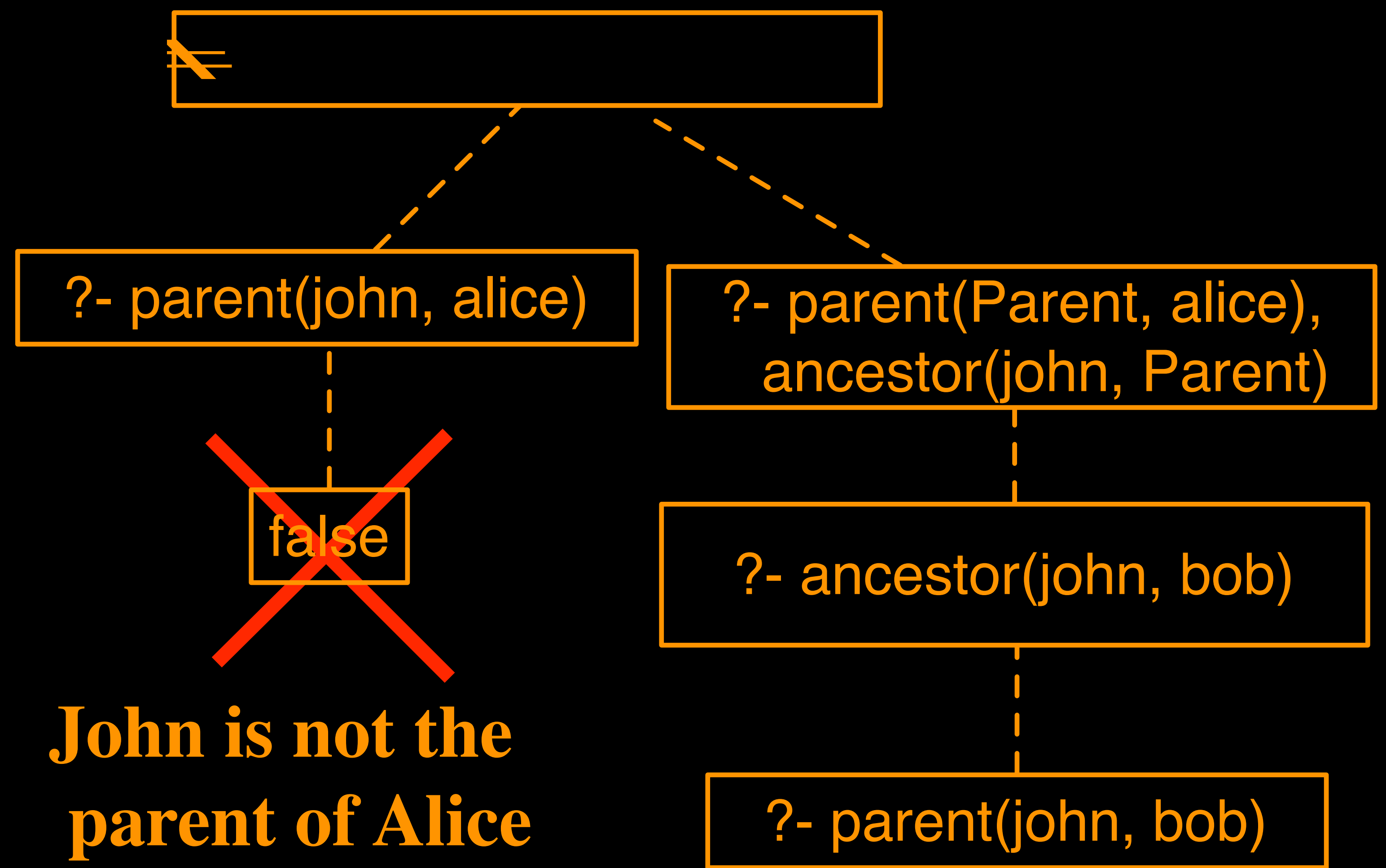

Bindings

```
Ancestor = john
Child = george
Parent = bob
```

Resolution

```
ancestor(Ancestor, Child) :-
        parent(Ancestor, Child)
ancestor(Ancestor, Child) :-
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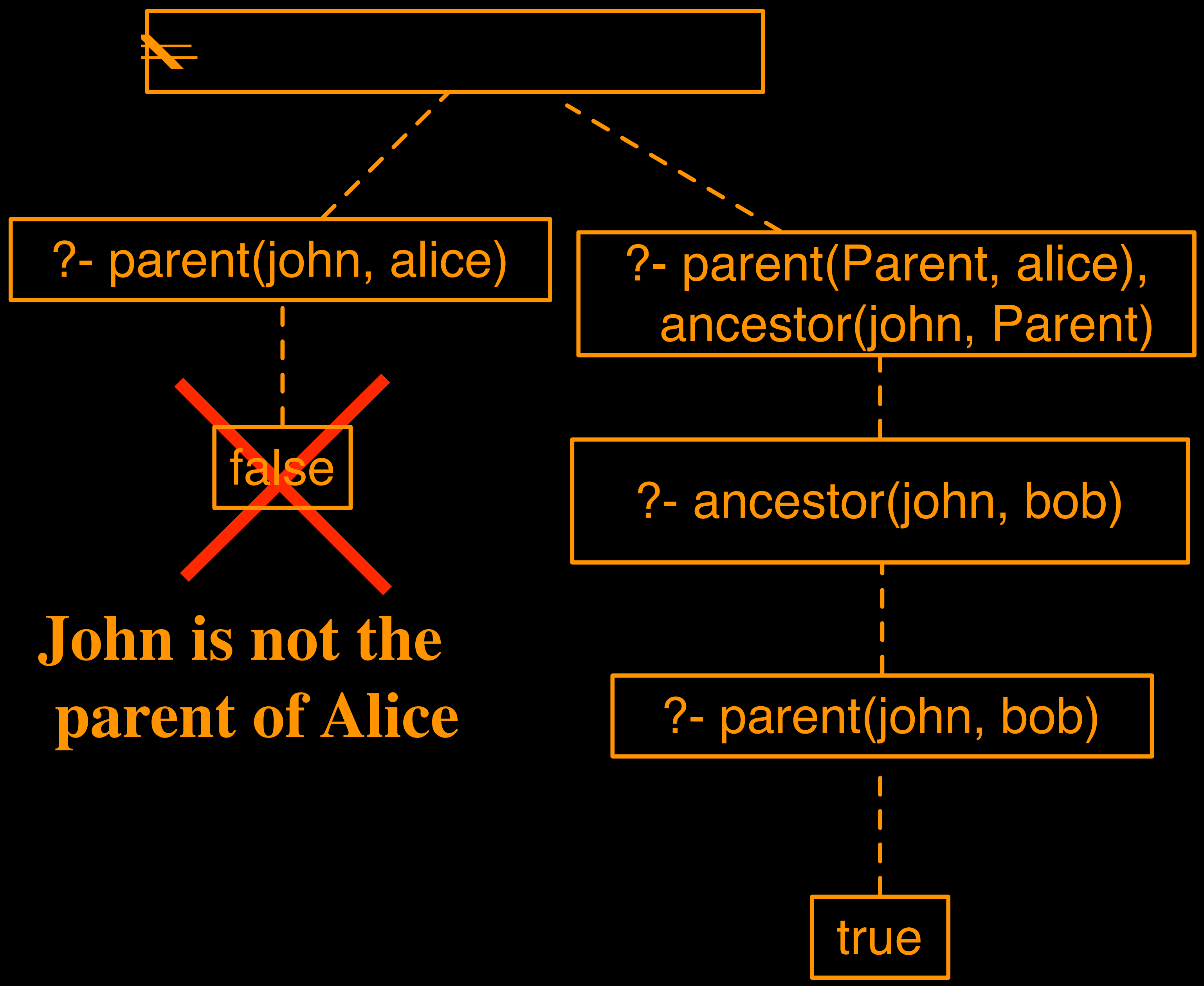


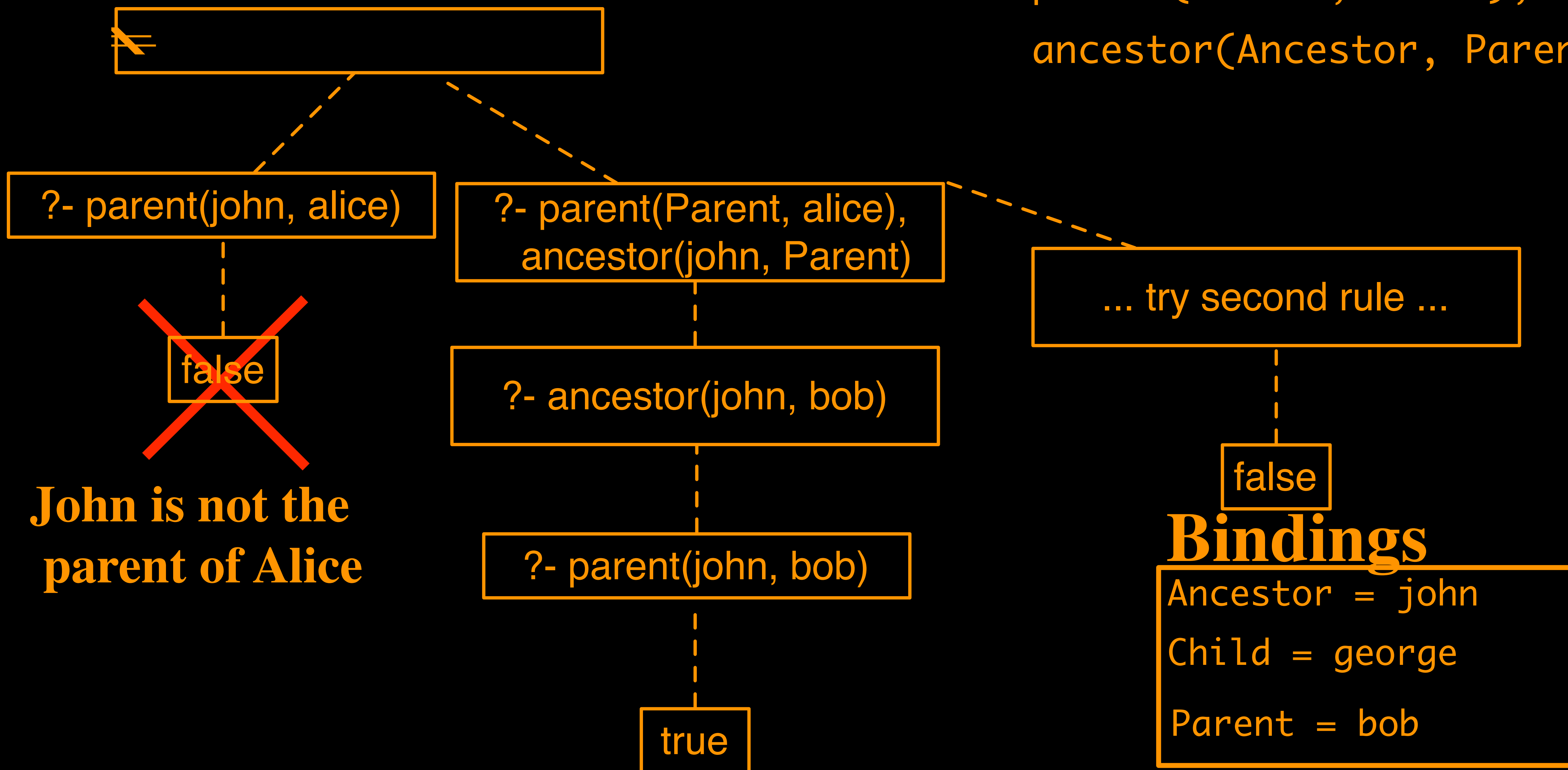
Bindings

```
Ancestor = john
Child = george
Parent = bob
```

Resolution

```
ancestor(Ancestor, Child) :-
      parent(Ancestor, Child)
ancestor(Ancestor, Child) :-
      parent(Parent, Child),
      ancestor(Ancestor, Parent)
```

- **How do queries work?**



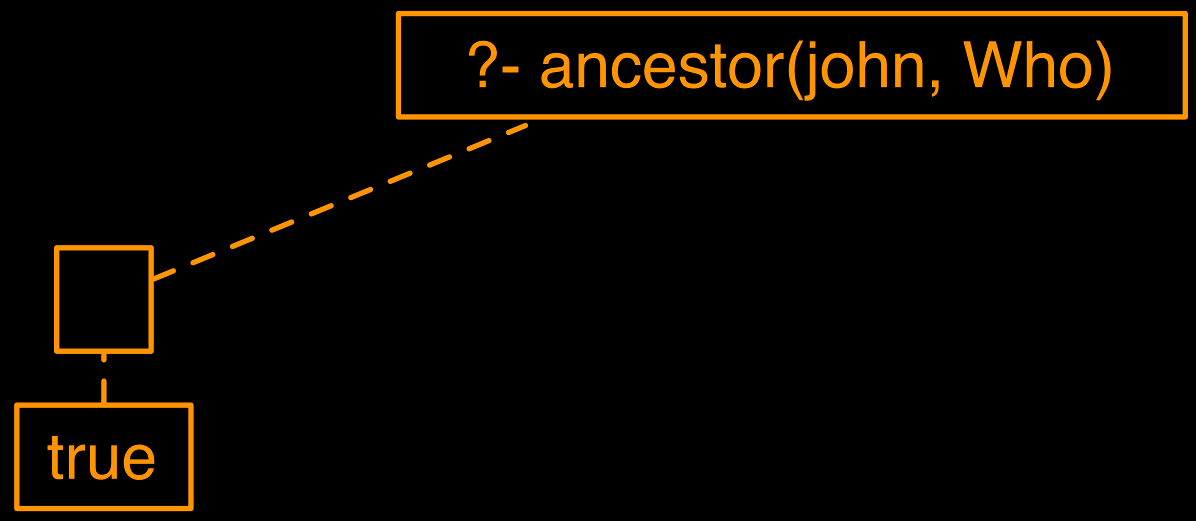
Resolution (2)

- **Start with the goal**
- **In order of definition:**
  - try each rule
  - unify head of rule with goal
  - try to solve the body of the rule (with substituted variables)
- **Depth-first**
  - First try one branch in the tree until you encounter a failure or a result
  - If that happens: backtrack to next place where another option was possible
- **Keep solving until no more possibilities**

Multiple results

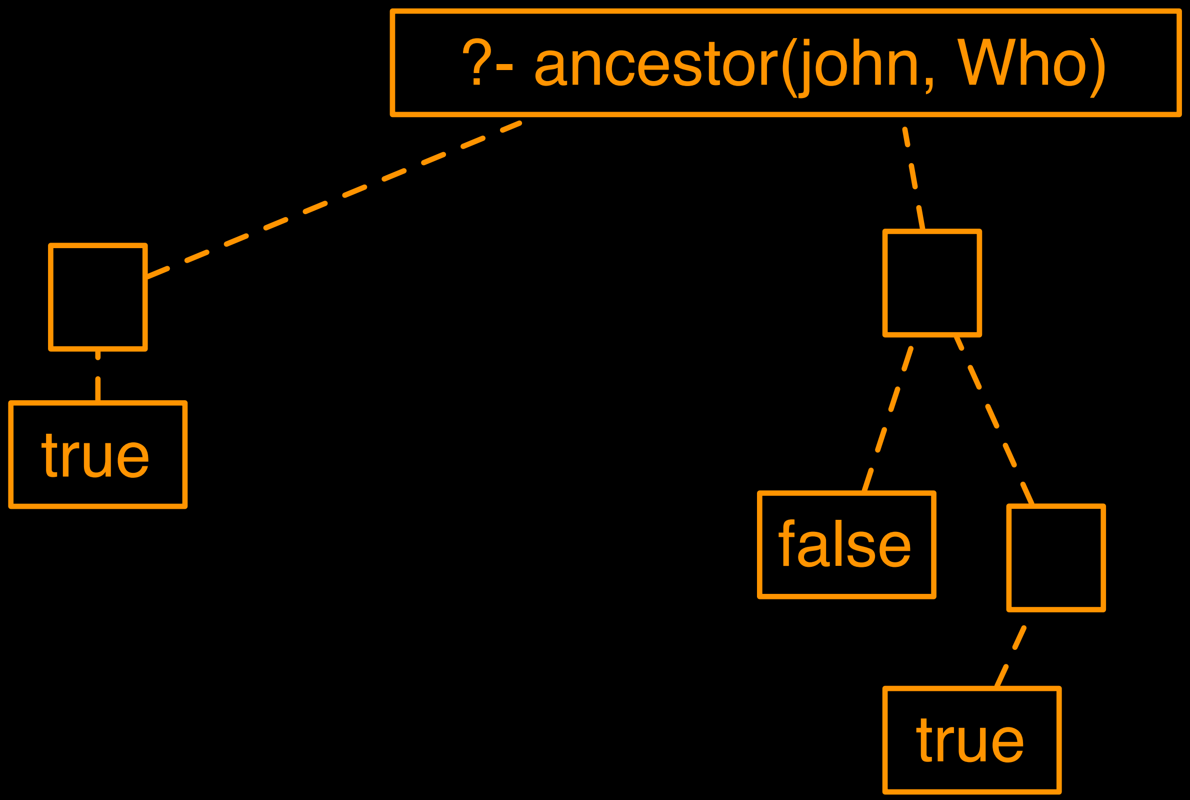
```
?- ancestor(john, Who)
```

Multiple results




```
Who = bob ;
```

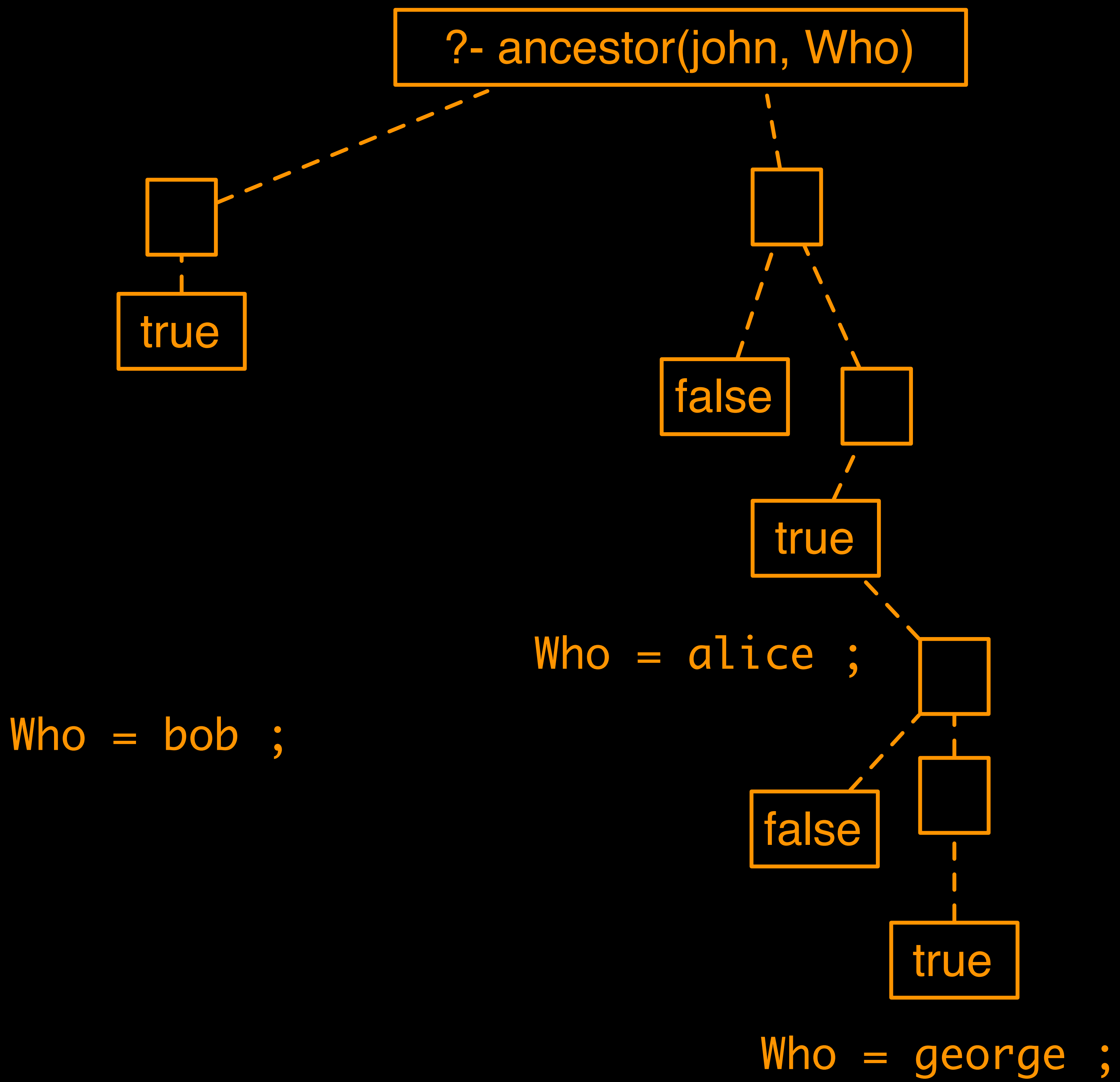
Multiple results



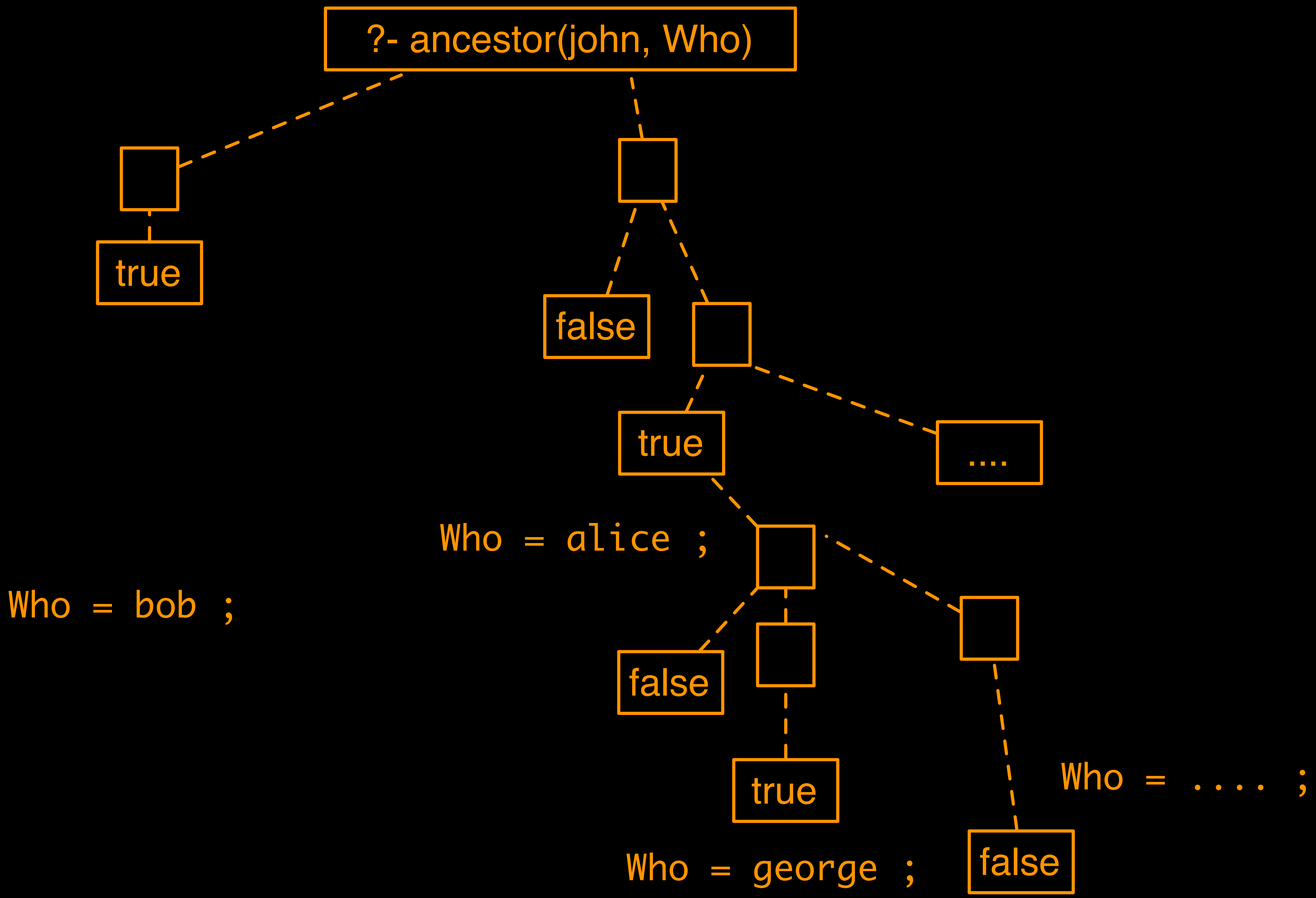

```
Who = alice ;
```

```
Who = bob ;
```

Multiple results



Multiple results



More Prolog: lists

Example of a lists: `[a, b, c, d, e, f]`

`[ ]`

In a rule:

```
[ A | B ]
```

Match the rest (B)

Match first element (A)

Applied to example:

```
A = a
B = [b, c, d, e, f]
```

Appending two lists

```
append([ ],List, List).

append([A | Rest], List, [A | Result] ) :-
     append(Rest, List, Result)
```

Remember: multi-way!

```
?- append([1,2,3],[4,5],[1,2,3,4,5]).

?- append([1,2,3],[4,5],List).

?- append([1,2,3],List,[1,2,3,4,5]).

?- append(List,[4,5],[1,2,3,4,5]).

?- append(List1,List2,[1,2,3,4,5]).
```

Negation as failure

- **Using `not` in rules and queries**
- **Try to prove the negated goal**
  - if success -> then fail
  - if fail -> then success
- **Closed-world assumption**
  - what is not known to be true, is false

```
bachelor(Person) :-
    male(Person),
    not(married(Person)

male(henry).
male(tom).

married(tom).
```

Negation (2)

```
?- bachelor(henry).
yes
?- bachelor(tom).
no
?- bachelor(Who).
Who= henry;

?- not(married(Who))
no
```

```
bachelor(Person) :-
   male(Person),
   not(married(Person)

male(henry).
male(tom).

married(tom).
```

Negation (2)

```
?- bachelor(henry).
yes
?- bachelor(tom).
no
?- bachelor(Who).
Who= henry;

?- not(married(Who))
no
```

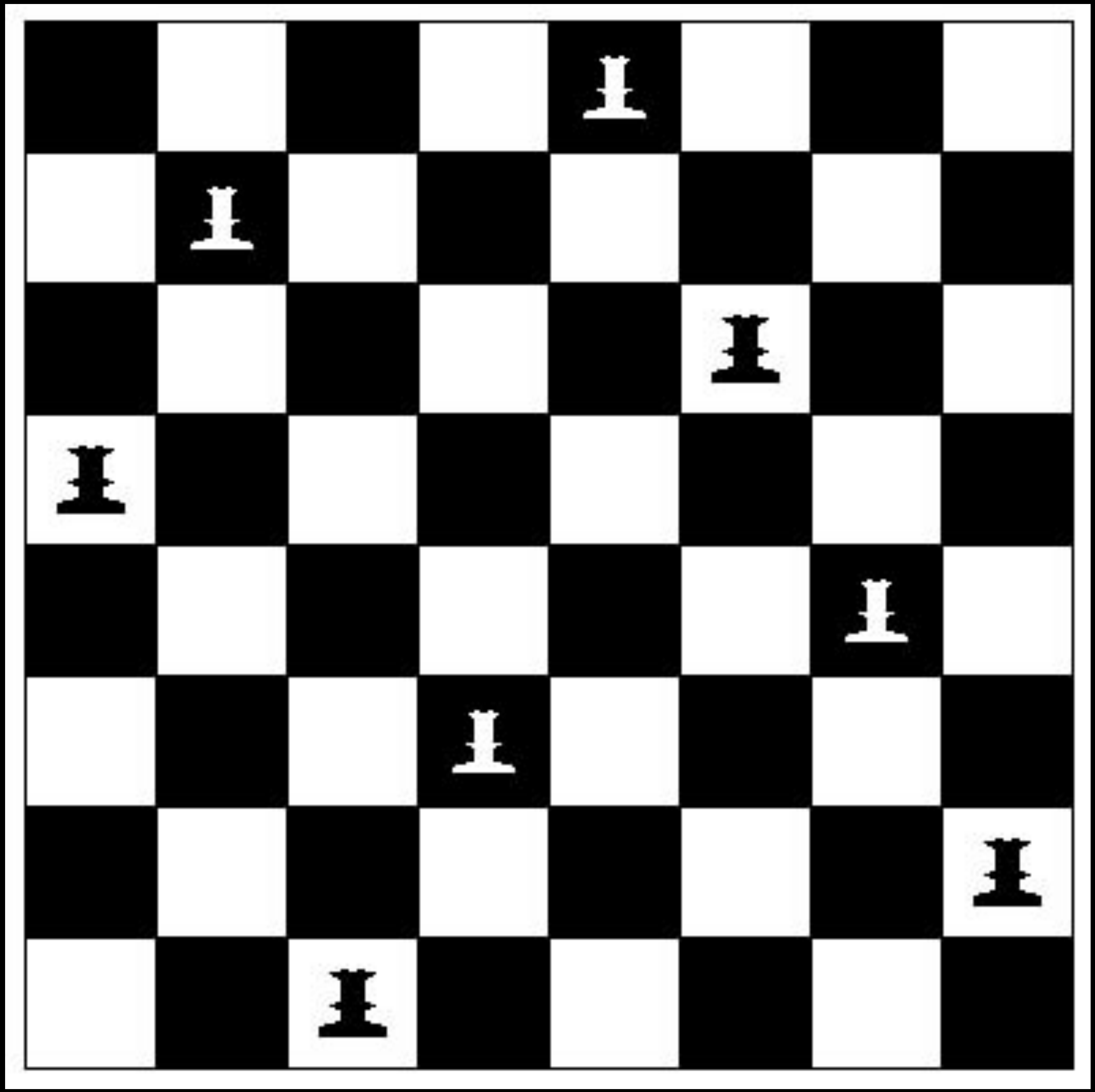
Why?

```
bachelor(Person) :-
   male(Person),
   not(married(Person))

male(henry).
male(tom).

married(tom).
```

8-queens problem



Place 8 queens on a chess board.

They should not be able to attack each other.

8-queens problem in Prolog

```
solutiontemplate([
    pos(1,Y1),
    pos(2,Y2),
    pos(3,Y3),
    pos(4,Y4),
    pos(5,Y5),
    pos(6,Y6),
    pos(7,Y7),
    pos(8,Y8)]).
```

Template for reporting the solution.

Given the column, calculate the row.

8-queens problem in Prolog (2)

```
solution8queens([]).
solution8queens([ pos(X,Y) | Others]) :-
        solution8queens(Others),
        member(Y, [1,2,3,4,5,6,7,8]),
        doesnotattack(pos(X,Y), Others).
```

Solve the problem recursively.

Try all possible Y positions.

Verify if it is a valid position.

8-queens problem in Prolog (3)

```
doesnotattack(pos(X,Y), []).
doesnotattack(pos(X,Y), [pos(X1,Y1) | Others]) :-
       not(Y = Y1),
       not(Y1 - Y = X1 - X),
       not(Y1 - Y = X - X1),
       doesnotattack(pos(X,Y), Others).
```

Valid if it does not attack any queen in the list of positions.

8-queens problem in Prolog (4)

```
?- solutiontemplate(S), solution8queens(S).

S = [pos(1, 7), pos(2, 8), pos(3, 5), pos(4, 6), pos(5, 4), pos(6, 3), pos(7, 2), pos(8, 1)] ;
S = [pos(1, 5), pos(2, 8), pos(3, 7), pos(4, 6), pos(5, 4), pos(6, 3), pos(7, 2), pos(8, 1)] ;
S = [pos(1, 7), pos(2, 5), pos(3, 8), pos(4, 6), pos(5, 4), pos(6, 3), pos(7, 2), pos(8, 1)] ;
S = [pos(1, 6), pos(2, 8), pos(3, 5), pos(4, 7), pos(5, 4), pos(6, 3), pos(7, 2), pos(8, 1)] ;
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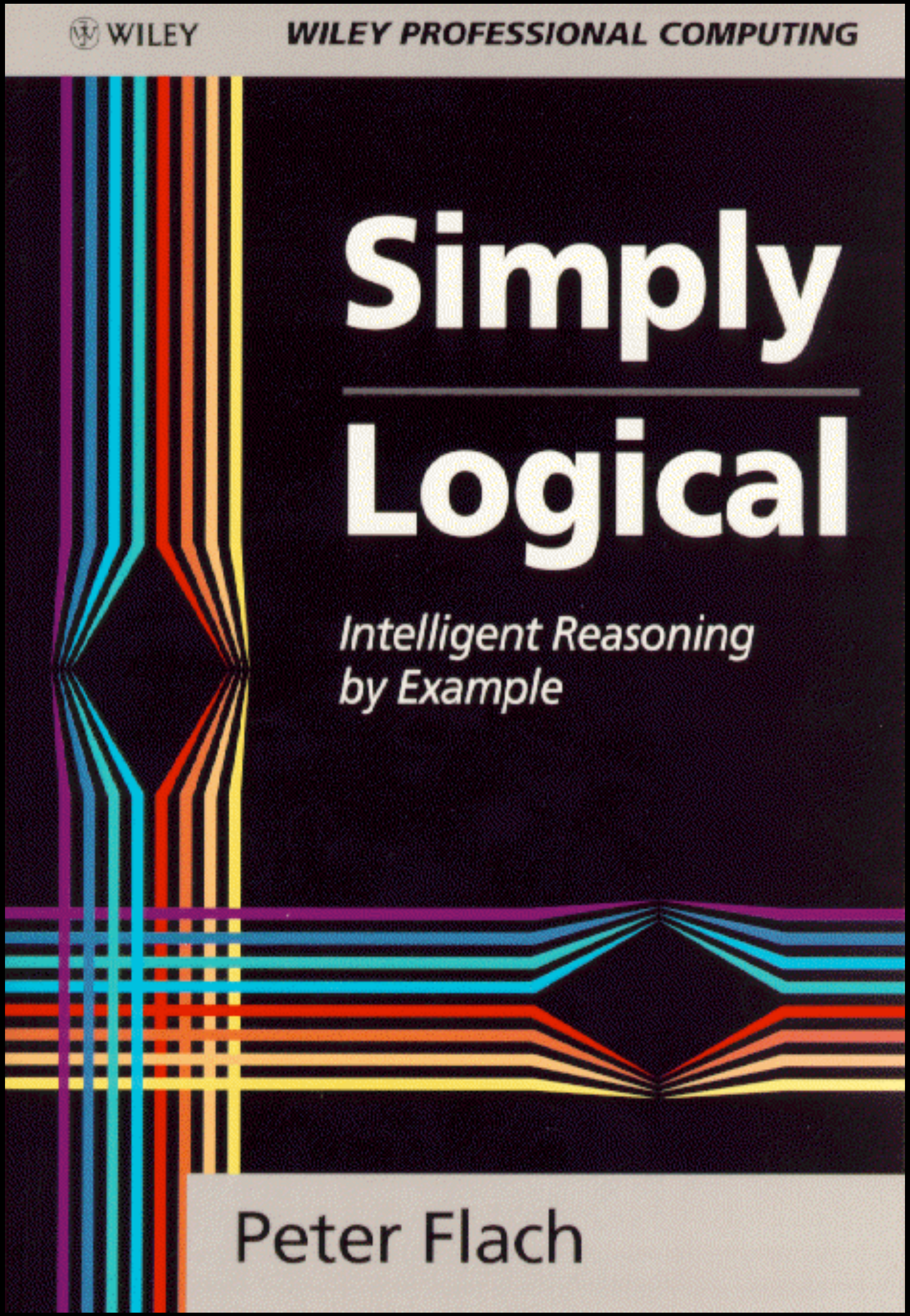
S = [pos(1, 7), pos(2, 8), pos(3, 4), pos(4, 6), pos(5, 5), pos(6, 3), pos(7, 2), pos(8, 1)] ;
S = [pos(1, 4), pos(2, 8), pos(3, 7), pos(4, 6), pos(5, 5), pos(6, 3), pos(7, 2), pos(8, 1)] ;
S = [pos(1, 7), pos(2, 4), pos(3, 8), pos(4, 6), pos(5, 5), pos(6, 3), pos(7, 2), pos(8, 1)] ;
S = [pos(1, 6), pos(2, 8), pos(3, 4), pos(4, 7), pos(5, 5), pos(6, 3), pos(7, 2), pos(8, 1)] ;
S = [pos(1, 6), pos(2, 4), pos(3, 8), pos(4, 7), pos(5, 5), pos(6, 3), pos(7, 2), pos(8, 1)] ;
S = [pos(1, 4), pos(2, 6), pos(3, 8), pos(4, 7), pos(5, 5), pos(6, 3), pos(7, 2), pos(8, 1)] ;
........
```

To conclude

- **Logic programming**
  - logic + control
  - unification/resolution
  - specify what you want, not how to calculate it!
- **Only scratched the surface**
  - Formal foundations
  - More language features (cut, meta programming, ...)
- **Lots of information out there!**

Free book!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